Nonintegrable Stadium Billiard seen as Mirror Cabinet

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The Bunimovich stadium billiard is treated as a mirror cabinet seen from inside. The resulting system of virtual images of the mirror walls reveals a fractional structure which corresponds to the chaotic trajectories of light rays or billiard balls inside the stadium.

Billiards of non-rectangular shape like the Bunimovich stadium (two semicircles separated by a square) are known to have a defocussing effect on billiard ball orbits. This results in "ray" trajectories which can be classified into a discrete set of periodic orbits of measure zero separated by a non-enumerable number of chaotic orbits [1, 2]. For an investigation of this complex dynamics a visual comparison of all trajectories inside the billiard is not suitable. It would be desirable to find a tool for reducing this great amount of information by mapping the dependence of the dynamic behavior on the initial conditions into a single

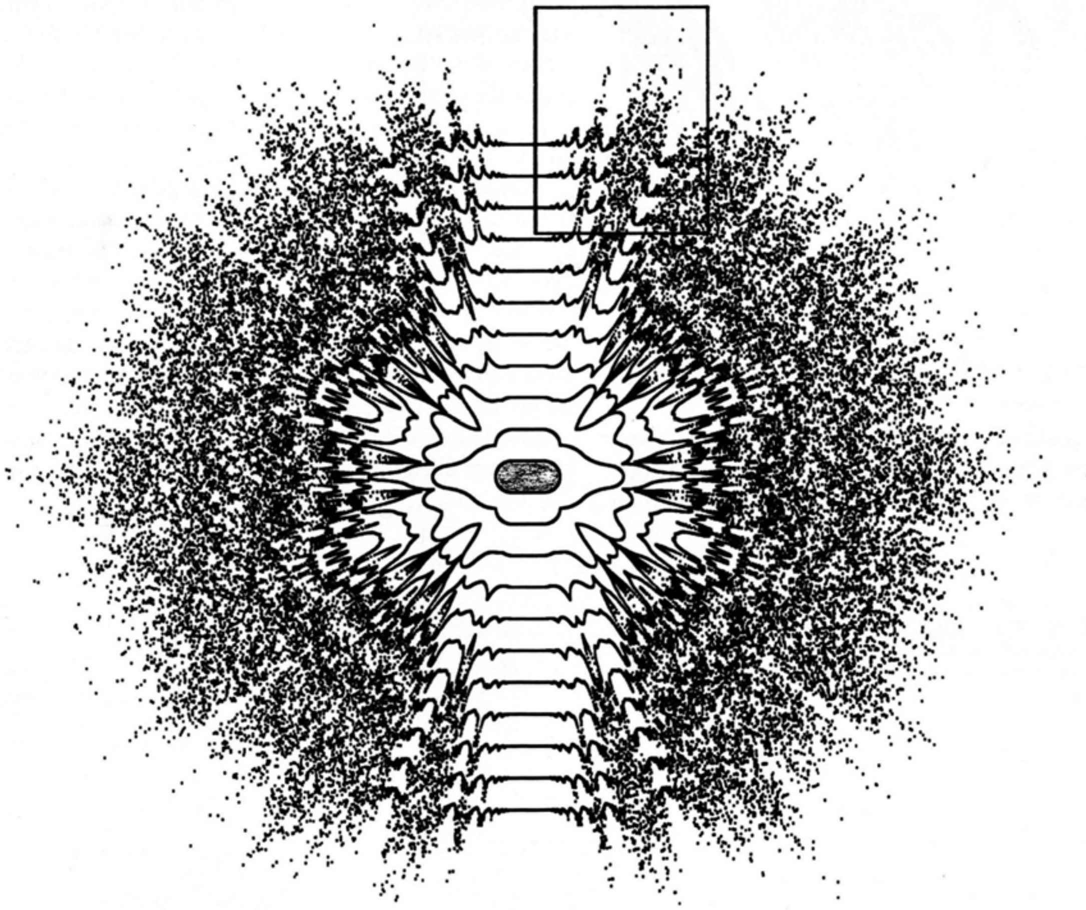

Fig. 1. Virtual image boundaries of a mirror cabinet with Bunimovich stadium shape (shaded area) seen from its center.

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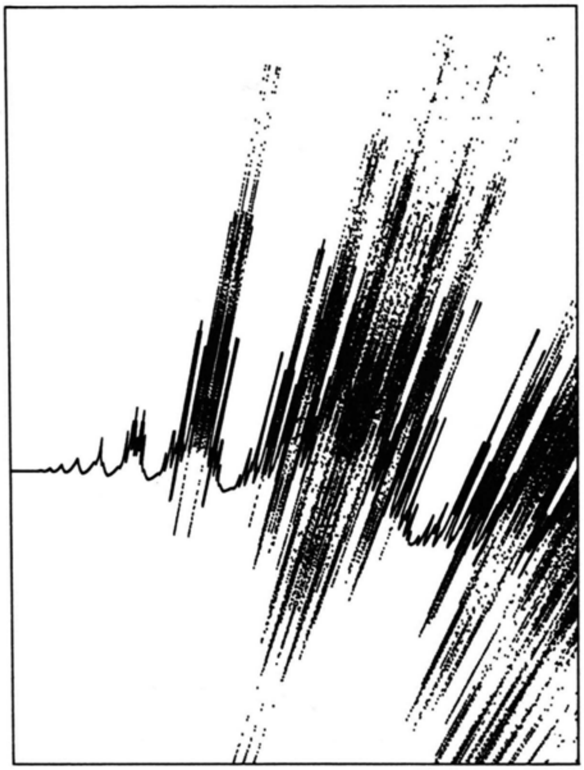

Fig. 2. Enlarged section of the 10th virtual image boundary.

diagram. Light rays propagate like billiard balls along trajectories governed by specular reflections. Therefore one can tackle this task by asking what does an observer inside a mirror cabinet of stadium shape see if the boundary is marked by a black line on the floor.

For solving this problem we consider a visual ray starting from an observation point inside the mirror cabinet and propagating into a chosen direction. Using a ray tracing technique we determine the path lengths between two consecutive reflections. The obtained sequence of path lengths is plotted along the visual ray in the initial direction ignoring the change of direction by the reflections at the boundary. If the same procedure is applied for all initial directions we finally get the virtual images of the mirror cabinet boundaries.

Figure 1 shows the mirror cabinet (shaded field) and the first 10 virtual image boundaries seen from the center of the Bunimovich stadium. The sensitive dependence of the ray trajectories on the initial conditions is correlated with the extreme roughness of the image boundaries which can be seen in the enlarged section of the 10th virtual image boundary (Figure 2). If the same algorithm is applied to mirror cabinets with shapes which are known to possess a non-chaotic dynamics (e.g. square, circle, quarter circle), the image boundaries remain smooth for all orders of reflection. Therefore our mapping technique serves as an example for the close connection of chaotic dynamics and fractal geometry in nature.

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[1] M. V. Berry, Proc. Roy. Soc. London A **413,** 183 (1987).
[2] M. C. Gutzwiller, Chaos in Classical and Quantum Mechanics, Springer-Verlag, New York 1990.